

# The Spectral-Domain Approach for Microwave Integrated Circuits

ROLF H. JANSEN, SENIOR MEMBER, IEEE

(*Invited Paper*)

**Abstract**—A survey is given of the so-called spectral-domain approach, an analytical and numerical technique particularly suited for the solution of boundary-value problems in microwave and millimeter-wave integrated circuits. The mathematical formulation of the analytical part of this approach is described in a generalized notation for two- and three-dimensional strip- and slot-type fields. In a similar way, the numerical part of the technique is treated, keeping always in touch with the mathematical and physical background, as well as with the respective microwave applications. A discussion of different specific aspects of the approach is presented and outlines the peculiarities of shielded-, covered-, and open-type problems, followed by a brief review of the progress achieved in the last decade (1975–1984). The survey closes with considerations on numerical efficiency, demonstrating that spectral-domain computations can be speeded up remarkably by analytical preprocessing. The presented material is based on ten years of active involvement by the author in the field and reveals a variety of contributions by West German researchers previously not known to the international microwave community.

## I. INTRODUCTION

GENERALLY SPEAKING, the term spectral-domain approach (SDA) refers to the application of integral transforms, such as the Fourier and Hankel transforms, to the solution of boundary-value and initial-value problems. As becomes obvious from the overview book and associated bibliography by Sneddon [1], this approach has been applied to mechanical and electromagnetic problems for at least a century. It provides an elegant tool for the reduction of the partial differential equations of mathematical physics to ordinary ones, which in many cases are amenable to further analytical processing. During the last 15 years, the spectral-domain approach has received considerably more interest together with the growing importance of printed circuits for very high frequencies, namely conventional and monolithic microwave and millimeter-wave integrated circuits (MIC's) fabricated by planar photolithographic technology. The actual and potential range of application of this technique implies hybrid thin- and thick-film circuits, monolithic MIC's on gallium arsenide, planar resonators and antennas, as well as multiconductor multilayer interconnections in high-speed computers. These circuits and components typically operate at frequencies between 0.1 and 100 GHz, and the main intention of using the approach has been the derivation of accurate, particularly frequency-dependent, design information.

Already by 1957, Wu [2] had considered it an “obvious thing to do” to apply a Fourier transform in the analysis of microstrip lines. From the end of the 1960's on, several authors began to implement more and more of those steps which are characteristic for what today is denoted the spectral-domain approach for MIC's. Yamashita and Mittra [3], for example, solved Poisson's equation in the transform domain and computed microstrip line capacitance from a variational expression under application of Parseval's theorem. Denlinger [4] in the United States and Schmitt and Sarges [5] in West Germany both derived an approximate solution to the microstrip dispersion problem in terms of the transformed strip current density. Itoh and Mittra [6], on the other hand, applied a spectral-domain approach in essentially the form it is still used today to the computation of slotline dispersion characteristics. Two years later, the same authors explicitly used the notation “spectral domain approach” for the specific technique (Galerkin's method in the transform domain) employed in one of their microstrip contributions [7]. Recent analyses still follow the basic outlines of this technique and the notation has been adopted by the microwave community.

In the initial research phase, a variety of fundamental applications and modifications of the spectral-domain approach and related methods had been reported within a few years. Coupled microstrip dispersion and characteristic impedances were computed by Kowalski and Pregla [8] and by Krage and Haddad [9]. Also, guided higher order modes in open microstrip lines were treated by Van de Capelle and Luypaert [10]. Itoh and Mittra [11] extended the spectral-domain formalism to shielded microstrip lines while Jansen [12] treated the same problem making use of a least-square criterion instead of Galerkin's method in the final step of the solution. As a first application to microstrip discontinuity problems, Rahmat-Samii *et al.* performed a quite general static spectral-domain analysis [13]. The first full-wave analyses of hybrid-mode microstrip resonator problems were reported by Itoh [14] and by Jansen [15], [16] in 1974, including rectangular, disk, ring, and concentric coupled shapes. Along the guidelines having emerged in this way, the spectral-domain approach has been used extensively for the characterization and analysis of elementary structures frequently appearing in MIC's. These structures can be classified as conducting thin patterns in one or more interfaces of a multilayer stratified dielectric medium. Therefore, the associated electromag-

Manuscript received February 5, 1985; revised May 31, 1985.

The author is with the University of Duisburg, Department of Electrical Engineering, FB9/ATE, Bismarckstrasse 81, D-4100 Duisburg 1, West Germany.

netic boundary-value problems lend themselves ideally to an SDA treatment. The partial differential equations considered are mainly the wave equation or, where small dimensions compared to wavelength prevail, the Laplace and the Poisson equation. Specific problems frequently tackled by the spectral domain approach are:

- 1) the static or frequency-dependent characterization of printed microwave transmission lines (a two-dimensional electromagnetic field problem).
- 2) the static or frequency-dependent analysis of problems concerning strip and slot transmission-line discontinuities, junctions and resonators, respectively and patch antennas (three-dimensional electromagnetic fields).

The contribution given here outlines the basic features of the analytical formulation of the spectral-domain approach as it applies to the above-mentioned problems. It is shown how for printed planar structures of arbitrary connected and disconnected shape embedded in a multilayer dielectric medium a single closed-form integral equation emerges from the application of the analytical steps of the SDA. As a result of explicit construction of that portion of the solution which depends on the vertical coordinate, this integral equation comes out reduced by one dimension compared to the original partial differential equation. From the beginning of the analysis, a considerable reduction in complexity is achieved and reduces the expense for the subsequent numerical part of the approach. This provides one of the important arguments for the superiority of the spectral-domain approach compared to other techniques.

In a discussion of the numerical procedure usually employed to solve the derived integral equation, the peculiarities of eigenvalue-type and deterministic MIC problems are treated briefly. There are arguments to prefer a Galerkin solution with certain symmetry properties for the former, while the latter do not generally result in a symmetric, respectively, Hermitian system of equations. From the obtained solutions, most of the quantities required in the characterization and analysis of MIC's can be obtained directly in the transform domain. Only one of the methods recently applied to MIC's shares several of the advantages of the SDA: the differential-difference approach (DDA), also called the method of lines [17]. A comparison with this, therefore, deserves a brief discussion.

After presenting these general features, the different aspects of the spectral-domain approach are outlined which have to be considered for shielded structures, laterally open structures, and configurations which are completely open electromagnetically. A specific implementation of the approach recently developed for the systematic frequency-dependent analysis of discontinuities and junctions in MIC's is described. It is discussed further as to how the radiation condition can be incorporated into the SDA formulation by proper choice of the integration path prevailing for the basic integral equation. To round out the

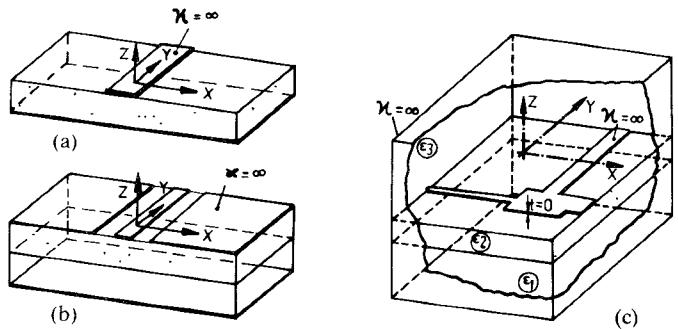


Fig. 1. (a) Microstrip line and (b) coplanar waveguide as examples for MIC transmission lines and (c) generalized MIC structure together with coordinate system used in the discussion.

picture given of the spectral-domain approach, the more important results achieved by its application are summarized in a subsequent section, and the state-of-the-art is described.

The last section of the paper is a discussion of the advantages and disadvantages of the spectral-domain approach. Emphasis is placed on the hybrid character of this technique which requires (and allows!) a certain amount of analytical preprocessing to achieve high efficiency. It is shown, further, how most of the disadvantages of the technique can be removed and to what degree, typically, a specific class of problems can be speeded up.

Remarks on the numerical problems associated with the development of user-oriented SDA packages are made and critical parts of these are illuminated. Finally, the main characteristics of the spectral-domain approach are summarized in a brief conclusion.

## II. MATHEMATICAL FORMULATION

Some elements of the analytical steps necessary to apply the spectral-domain approach to specific problems, particularly the characterization of MIC transmission lines, have already been described in overview books [18], [19]. The treatment given here generalizes the formulation as far as possible and emphasizes those features which the different classes of MIC problems all have in common. For a visualization of the physical construction of the configurations to be considered, Fig. 1 shows (a) an open microstrip line, (b) a coplanar waveguide suspended above the ground plane of a circuit environment, (c) and a quite general shielded structure. The latter serves for the following discussion and could as well be laterally open or completely open. It provides an idealized view of the basic construction of MIC's indicating that the passive portions of these consist mainly of a thin conductor metallization in one or more interfaces of a double or multilayer dielectric medium; for an overview, see [20].

In agreement with common microwave practice, the formulation is in terms of time-harmonic electromagnetic fields, namely, a time dependence of  $\exp(j\omega t)$ . Vector quantities, like the electric field  $\underline{E}$ , are written by single underlining, matrices by double underlining. The involved conductors are assumed to have ideal conductivity  $\kappa$  and

negligible thickness  $t$ . This is a very realistic assumption in hybrid MIC's, where strip and slot widths are usually large compared to conductor thickness. In monolithic MIC's on gallium-arsenide, this is not valid with the same generality. Here, the assumption is mainly a matter of convenience and simplification of the treatment. The consideration of finite thickness in the SDA formulation can be achieved by treating the thick metallization as a separate layer, see for example [21]–[23]. In addition, it is convenient in most cases to assume lossless dielectric media since this allows a real number arithmetic for the SDA algorithm, except for cases where radiation or surface-wave excitation is involved. Loss parameters are usually introduced by perturbation methods subsequent to a numerical solution neglecting loss. This also applies to the evaluation of conductor loss, which can be taken into account if the asymptotic behavior of the field derived for zero metallization thickness is appropriately modified [24]–[27]. In each of the layers  $i=1,2\dots L$  of a general configuration like that of Fig. 1, the electromagnetic field is best formulated in terms of scalar LSM and LSE wave potentials [28], [29]. This is equivalent to the use of vector wave potentials having only one component in the  $z$ -direction, i.e., perpendicular to the stratified circuit medium. It allows a completely decoupled and, therefore, particularly simple analytical treatment of all classes of MIC problems [30]–[36] independent of the number of dielectric layers involved. This specific choice naturally leads to what Itoh [35] has named the spectral-domain immittance approach. Just recently, Omar and Schünemann [37] have shown that only coupling of the LSE and the LSM contributions to the electromagnetic field occurs and is required in satisfying the edge condition with the last step of the solution. The scalar LSM and LSE wave potentials are denoted  $f$  and  $g$  here. They are subject to the homogeneous Helmholtz equation

$$(\Delta + k_i^2) f_i = (\Delta + k_i^2) g_i = 0 \quad (1)$$

in each of the layers  $i=1,2\dots L$  of the circuit medium, with  $k_i$  denoting the wave number associated with the  $i$ th layer. It should be stressed that the homogeneous form of (1) applies even in the case of excitation problems. With the spectral-domain approach, sources are introduced in a natural way as impressed current densities or electric fields only into the interfaces between the layers [38]–[41]. Instead of considering the space-domain form of the Helmholtz equation (1) directly, its spectral-domain equivalent is used. Without loss of generality, the scalar potentials may be written in the form of inverse two-dimensional Fourier transforms, for example,

$$f_i(x, y, z) = \int_{C_x} \int_{C_y} \tilde{f}_i(k_x, k_y, z) \exp(-j(k_x x + k_y y)) dx dy. \quad (2)$$

For configurations of circular symmetry, the use of Hankel transforms is an adequate choice [1], [42]–[46]. In transmission-line problems, (2) is reduced by one dimension since

these problems can be formulated in terms of the cross-sectional field alone postulating a longitudinal dependency of the form  $\exp(-jk_y y)$ . Alternatively, for these cases, the spectral wave potential  $\tilde{f}_i$  may be viewed as factorized and containing a  $y$ -dependent factor in the form of a Dirac distribution, so that the integration in (2) reduces to one dimension. In the high majority of SDA contributions published, the integration paths  $C_x$  and  $C_y$  have been chosen to coincide with the respective coordinate axes. The spectral variables  $k_x$  and  $k_y$  may be interpreted each as a measure of spatial oscillation of the described field which is useful for later convergence considerations. This is immediately obvious if (2) implies a finite Fourier transform [11] and the spectral wave numbers  $k_x$  and/or  $k_y$  form an infinite numerable sequence. In that case,  $f_i(k_{xm}, k_{yn}, z)$  describes the Fourier series coefficients of  $f_i(x, y, z)$  and these values of  $k_{xm}, k_{yn}$  are chosen in such a way that the boundary conditions on a lateral shielding parallel to the  $z$ -axis are satisfied. Further generalizing, one may consider such coefficients as being associated with any two suitably chosen complete orthogonal sets of solutions of the Helmholtz equation which are TM and TE with respect to the  $z$ -direction and satisfy the lateral boundary conditions [30], [31]. At the same time, this reveals how a suitable finite integral transform can be constructed for a given cross section of the shielding. In addition, this generalization makes clear that the mathematical formulation can be discussed completely independent of the special cross section or even the existence of a lateral shielding. The Helmholtz operator of (1), if applied in the transform domain, always appears as

$$\tilde{\Delta} + k_i^2 = \frac{\partial^2}{\partial z^2} + k_i^2 - k_x^2 - k_y^2 = \frac{\partial^2}{\partial z^2} + k_{zi}^2 \quad (3)$$

which is an ordinary differential operator. Due to the simple layered planar construction of MIC's, the transformed wave potentials  $\tilde{f}_i$  and  $\tilde{g}_i$  can be determined analytically and adopt the general form

$$\begin{aligned} \tilde{f}_i(k_x, k_y, z) &= a_i(k_x, k_y) \cdot \cos(k_{zi}(z - z_i)) \\ &+ b_i(k_x, k_y) \cdot \sin(k_{zi}(z - z_i)), \quad i = 1 \dots L. \end{aligned} \quad (4)$$

The functions  $a_i$  and  $b_i$  are spectral distributions weighting the elementary plane-wave constituents with respect to the  $z$ -axis. The parameter  $z_i$  is arbitrary and is, for example, introduced to allow convenient satisfaction of the boundary conditions at the conducting ground plane and cover shielding usually existing in MIC's. With the relations

$$j\omega\epsilon_i \tilde{E}_{zi} = k_p^2 \tilde{f}_i, \quad j\omega\mu_i \tilde{H}_{zi} = k_p^2 \tilde{g}_i, \quad k_p^2 = k_x^2 + k_y^2 \quad (5)$$

it becomes clear that homogeneous boundary conditions prevail for  $\tilde{f}_i$  and  $\tilde{g}_i$  identical to those for the transformed field components  $\tilde{E}_{zi}$  and  $\tilde{H}_{zi}$ . Further, since the transform defined by (2) affects only the  $x$  and  $y$  coordinates, all conditions specified for the spatial electromagnetic field in planes of constant values of  $z$  can be directly transferred into the spectral domain. Therefore, in a configuration involving the layers  $i=1,2\dots L$ , the potential  $\tilde{f}_i$  at the

ground and top plane of the multilayer medium is

$$\tilde{f}_i(k_x, k_y, z) = a_i(k_x, k_y) \cos(k_{zi}(z - z_i)), \quad i = 1, L \quad (6)$$

if  $z_1$  and  $z_L$  describe the positions of the ground and cover shielding. In complete analogy,  $\tilde{g}_i$  is proportional to the respective sine function for  $i = 1$  and  $i = L$  as a consequence of the vanishing of  $H_{zi}(x, y, z)$  at  $z = z_1$  and  $z = z_L$ . For vertically open structures like antennas and open microstrip, the potentials  $\tilde{f}_L$  and  $\tilde{g}_L$  both have an exponential  $z$ -dependence. The complete transformed electromagnetic field in all of the layers  $i = 1, 2, \dots, L$  is derived by application of the spectral-domain equivalent  $\tilde{\nabla}$  of the  $\nabla$  operator, namely, by

$$\begin{aligned} \underline{\tilde{E}}_i &= \frac{1}{j\omega\epsilon_i} \tilde{\nabla} \times \tilde{\nabla} \times (\tilde{f}_i \underline{u}_z) - \tilde{\nabla} \times (\tilde{g}_i \underline{u}_z) \\ \underline{\tilde{H}}_i &= \frac{1}{j\omega\mu_i} \tilde{\nabla} \times \tilde{\nabla} \times (\tilde{g}_i \underline{u}_z) + \tilde{\nabla} \times (\tilde{f}_i \underline{u}_z). \end{aligned} \quad (7)$$

These relations have the same form as their spatial counterparts [28], [29] and result by substituting the algebraic multiplicators  $jk_x$  and  $jk_y$  for the respective partial differential operators. Together with the foregoing discussion they show that, in a circuit medium of  $L$  layers, the total electromagnetic field can be described by  $4(L-1)$  independent spectral LSM and LSE distributions  $a_i(k_x, k_y)$ ,  $b_i(k_x, k_y)$ . For the dielectric interfaces between the different layers, exactly the same number of continuity conditions can be formulated in the spectral domain, i.e.,

$$\begin{aligned} \tilde{E}_{xi} - \tilde{E}_{xi+1} &= 0 & \tilde{E}_{yi} - \tilde{E}_{yi+1} &= 0 \\ \tilde{H}_{xi} - \tilde{H}_{xi+1} &= -\tilde{J}_{yi} & \tilde{H}_{yi} - \tilde{H}_{yi+1} &= +\tilde{J}_{xi} \end{aligned} \quad (8)$$

for  $i = 1, \dots, L-1$  and  $z$  fixed at its interface value for each subscript  $i$ . They mirror the continuity of the electric field  $\underline{\tilde{E}}_i$  tangential to the interfaces independent of the presence of a thin metallization. At the same time, they describe the discontinuity of the tangential magnetic field at such a metallization in terms of a surface current density  $\underline{J}_i$ . In interfaces which do not contain conductors,  $\underline{J}_i$  is defined to be zero, enforcing the magnetic field continuity there. By analytical processing of the relations (8), all the unknown distributions  $a_i$  and  $b_i$  can be eliminated or expressed by the spectral-domain current density components  $\tilde{J}_{xi}$  and  $\tilde{J}_{yi}$ . The latter may exist in only one of the interfaces or in several of these. In this stage of the analysis, the only boundary conditions which remain to be satisfied are those of the electric field  $\underline{\tilde{E}}_i$  tangential to the conductor metallization and of the surface current density  $\underline{J}_i$  to vanish in complementary regions. How the spectral-domain relations resulting from the analytical evaluation of (8) have to be arranged for further processing, therefore, depends on whether the metallized interfaces can be characterized as strip-type ( $i = ist$ ) or slot-type ( $i = isl$ ), respectively. In the general case, an algebraic spectral-domain

equation results linking the mixed-type vectors

$$(\dots \tilde{E}_{xist}, \tilde{E}_{yist} \dots \tilde{J}_{xisl}, \tilde{J}_{yisl} \dots)^T$$

and

$$(\dots \tilde{J}_{xist}, \tilde{J}_{yist} \dots \tilde{E}_{xist}, \tilde{E}_{yist} \dots)^T \quad (9)$$

by a spectral immittance matrix; see for example [22], [35], [47].  $T$  denotes transposition and is used only for convenience of writing. The lower one of the two vectors shown is put onto the right side of the described algebraic relation because its elements are better suited for expansion into known functions. These elements are typically confined to a small portion of the affected interfaces. For a similar reason, the upper one of the vectors is arranged onto the left side of the spectral-domain equation. By this, it is described by the other vector and needs to satisfy boundary conditions on only a small portion of its region of existence. The spectral algebraic relation between the vectors (9) is equivalent to a single integral equation which results by application of the transform inherent in (2). Since the whole discussion has been performed without recourse to particular shapes of the metallization pattern, this is true for arbitrary planar connected and disconnected conductors. From the procedure outlined, the kernel of the integral equation is available in analytical form. Further, this integral equation comes out reduced by one dimension compared to the original field problem associated with (1). It is one-dimensional for transmission-line problems and two-dimensional for discontinuities, resonators, and so on. As (4) shows, the  $z$ -dependency of the field is described in analytical form. In most cases of practical interest, the MIC configurations analyzed are either strip-type or slot-type exclusively, with only one layer of metallization. Under this presumption, the relation between the vectors (9) reduces to the simpler form

$$(\tilde{E}_{xist}, \tilde{E}_{yist})^T = \underline{\tilde{Z}}(p) \cdot (\tilde{J}_{xist}, \tilde{J}_{yist})^T$$

or

$$(\tilde{J}_{xist}, \tilde{J}_{yist})^T = \underline{\tilde{Y}}(p) \cdot (\tilde{E}_{xist}, \tilde{E}_{yist})^T. \quad (10)$$

The quantity  $p$  has been introduced to remind one of the fact that the elements of  $\underline{\tilde{Z}}$  and  $\underline{\tilde{Y}}$  depend in a known, analytical form on physical parameters defining the considered MIC problem (for example, vertical geometry, shielding dimensions, or operating frequency). Obviously, the spectral immittance matrix  $\underline{\tilde{Y}}$  is the inverse of  $\underline{\tilde{Z}}$  if both equations in (10) refer to the same problem and dielectric interface. For this reason, we may also write in the space domain

$$\underline{E}_i = L_\infty(p) \cdot \underline{J}_i \quad \text{or} \quad \underline{J}_i = L_\infty^{-1}(p) \cdot \underline{E}_i \quad (11)$$

where  $\underline{E}_i = (E_x, E_y)^T$  and  $\underline{J}_i = (J_x, J_y)^T$  are specialized to denote electric field and current density in the plane of the circuit metallization. The integral operators  $L_\infty$  and  $L_\infty^{-1}$  are linear with respect to the vectors they operate on and both have a dyadic kernel defined by the spectral-domain Green's immittance matrices  $\underline{\tilde{Z}}$  and  $\underline{\tilde{Y}}$ . The constituents of

this kernel can be determined in a particularly elegant way by the so-called spectral-domain immittance approach [32], [35]. It has been shown by the author for shielded configurations that by suitable choice of an orthonormal vectorial function basis in (2), the kernel can be described by a single infinite scalar set of wave impedances  $\tilde{Z}_n$  in conjunction with the elements of the function basis [30], [31]. In that case, the discrete impedance elements  $\tilde{Z}_n$  are equal to the modal input wave impedances of the cylindrical shielding as seen in the plane of metallization. Independent of these details, there is always a duality between the strip-type and the slot-type formulation as visible in (10) and (11). For the former, the tangential electric field  $\underline{E}_t$  has to vanish on the strip metallization  $F_{st}$ , while for the latter, the surface current density  $\underline{J}_t$  does not exist in slot regions  $F_{sl}$ , i.e., outside of the metallization. Splitting up the right-hand quantities of (11) into an excited term (subscript  $tex$ ) and a source or impressed term (subscript  $tim$ ) further yields

$$\begin{aligned}\underline{E}_t &= \underline{0} = L_{\infty}(p) \cdot (\underline{J}_{t ex} + \underline{J}_{t im}) \text{ for } x, y \in F_{st} \\ \underline{J}_t &= \underline{0} = L_{\infty}^{-1}(p) \cdot (\underline{E}_{t ex} + \underline{E}_{t im}) \text{ for } x, y \in F_{sl}.\end{aligned}\quad (12)$$

These final integral equations are written here as in the formulation of a scattering problem [38]–[41]. They define, at the same time, the electric field  $\underline{E}_t$  and current density  $\underline{J}_t$  in the complementary regions  $F_{st}^{-1}$  and  $F_{sl}^{-1}$ , i.e., outside of strips and on the metallization around slots. If sources  $\underline{J}_{t im}$ ,  $\underline{E}_{t im}$  are not present, like in transmission-line and resonator problems, the equations in (12) each define a so-called nonstandard eigenvalue problem [48]–[50]. This notation applies since, without sources, (12) can be solved only for specific values of the parameter  $p$  (the eigenvalue), which is contained in the integral equation kernels in nonlinear, usually transcendental form. Which physical quantity is chosen in a problem as the parameter  $p$  is to some degree arbitrary. In transmission-line problems, the usual choice is  $p = \beta$ , i.e., the propagation constant, or  $p = \beta^2$ , the square of it. Resonator problems are conveniently treated in terms of  $p = \omega_0$ , the resonance frequency, or  $p = l_0$ , a dimension of the resonator.

### III. NUMERICAL SOLUTION

The standard procedure applied in most computer solutions of the integral equations (12) today is Galerkin's method in the spectral domain, particularly for the eigenvalue problem. This is a preferable choice resulting from the self-adjointness of the involved integral operators and following the argumentation by Harrington [51]. The stationarity of such solutions has been discussed in an early contribution by the author in comparison to a least-squares alternative [31]. It has recently been shown by Lindell in a general context for the eigenvalue parameter  $p$  with respect to the trial field [48]–[50] which is  $\underline{J}_{t ex}$  for strip problems and  $\underline{E}_{t ex}$  for slot-type configurations. To simplify the discussion, restriction to strip-type problems is allowed without loss in generality. The numerical procedure is best understood if the equations prevailing in the

spectral and the space domain are considered in parallel, i.e., writing briefly

$$\underline{E}_t = L_{\infty}(p) \cdot (\underline{J}_{t ex} + \underline{J}_{t im}) \quad \tilde{\underline{E}}_t = \tilde{Z}(p) \cdot (\tilde{\underline{J}}_{t ex} + \tilde{\underline{J}}_{t im}). \quad (13)$$

In the space domain, the physical vectors  $\underline{E}_t$  and  $\underline{J}_t = \underline{J}_{t ex} + \underline{J}_{t im}$  are different from zero in the complementary regions  $F_{st}^{-1}$  and  $F_{sl}^{-1}$ . The unknown surface current density  $\underline{J}_{t ex}$  is expanded into a suitable, preferably complete set of expansion functions defined on  $F_{st}$  and vanishing outside. By this, continuity of the magnetic field outside the metallization is achieved at the same time. The expansion of  $\underline{J}_{t ex}$  is actually performed in the original, spatial domain since this provides the best physical insight for a good choice. It depends on the specific problem under investigation whether the functions  $\underline{J}_{tk}$  chosen should be easily transformable into the spectral domain or not. For the application of Galerkin's method, the set of testing functions necessary to enforce the vanishing of  $\underline{E}_t$  on the conductor region  $F_{st}$  is the same as the expansion used, say  $\underline{J}_{tj}$ . The scalar product employed in the testing process is commonly defined by integration over  $F_{st}$  without a specific weighting factor and expressed here using parentheses  $(\cdot)$ . Making use of the linearity of the integral operator involved, the standard process of testing [51] finally yields

$$\sum_k \alpha_k (\underline{J}_{tj}, L_{\infty}(p) \cdot \underline{J}_{tk}) = -(\underline{J}_{tj}, L_{\infty}(p) \cdot \underline{J}_{t im})$$

or

$$\sum_k \alpha_k (\tilde{\underline{J}}_{tj}, \tilde{Z}(p) \cdot \tilde{\underline{J}}_{tk}) = -(\tilde{\underline{J}}_{tj}, \tilde{Z}(p) \cdot \tilde{\underline{J}}_{t im}). \quad (14)$$

The second alternative and mathematically identical equation applies as a consequence of Parseval's theorem [1], which also serves for a unique definition of the associated scalar product  $(\cdot)$  in the spectral domain. In eigenvalue problems, the right-hand side of (14) vanishes and a non-trivial solution  $\underline{J}_{t ex}$  exists only if the determinant of the respective linear system of equations is zero. This provides the nonlinear eigenvalue equation for the unknown parameter  $p$  and, subsequent to an iterative evaluation of  $p$ , the associated surface current distribution  $\underline{J}_t = \underline{J}_{t ex}$ . The electric field outside of the metallization is found from the application of (13). For MIC excitation problems, (14) is deterministic and can be solved in a single step for a prescribed value of the parameter  $p$ . The main difficulty in such cases is a realistic formulation of the excitation term  $\underline{J}_{t im}$  such that it well describes the physical situation. Also, the introduction of such a source term may complicate the satisfaction of boundary conditions in its spatial vicinity as compared to an equivalent eigenvalue formulation [36]. As long as the source chosen has finite support in the  $x$ - $y$  plane, the field region is finite and the expansion functions are chosen properly, Galerkin's method can still be applied, for example, if the source is a slit voltage generator or a strip current sheet [38], [39]. However, if the field is excited by a transmission-line mode coming in from infinity and a reflected wave is involved, Galerkin's method

cannot be applied any more since the associated scalar products become unbounded. In that case, which is a good description of practical MIC excitation problems, another version of moment methods has to be employed [40], [41] enforcing existence of the scalar products. Expansion and test functions have to be different then with the consequence that the final system of equations (14) is not symmetric or Hermitian any longer.

It should have become obvious from the discussion that interpreting the numerical procedure as "Galerkin's method in the transform domain" is too restrictive not only because of the last-mentioned details. Actually, it does not make a mathematical difference which one of the equations (14) is considered if the presumptions necessary for the application of Parseval's theorem are satisfied. As a rule of thumb, in laterally open problems, evaluation of the scalar products in (14) is alleviated if the spectral quantities are used. In these cases, expansion functions should be selected with explicitly available analytical transforms. On the other hand, for shielded configurations, it may have advantages to perform the scalar product operation in the spatial domain, particularly if a suitable orthogonal set of functions can be constructed for the description of the electromagnetic field [30], [31]. Also, it is generally easier to construct complete sets of expansion functions for conductors of complex shape in the space domain. So, the major advantage of the so-called spectral-domain approach is that it allows one to shift between the space and the transform domains in essentially all steps of the processing. The same applies to the computation of MIC design quantities from solutions obtained by the approach. Quantities like quality factors, dielectric and magnetic loss, conductor loss, and power transported in the cross section of transmission lines can with advantage be computed in either of the domains depending on the shielding situation and the specific problem. The evaluation of such design data involves volume or surface integration over the products

$$\underline{E}_i \cdot \underline{E}_i^* \quad \underline{H}_i \cdot \underline{H}_i^* \text{ and } \underline{E}_i \times \underline{H}_i \quad (15)$$

where the asterisk denotes complex conjugate. Integration over the vertical  $z$ -coordinate is always performed analytically due to the simple trigonometric dependencies associated with the layered MIC structure. Along the other coordinates, Parseval's theorem again allows a choice. Care has to be taken in conductor loss computations for metallizations of zero thickness. Because of the order of the edge singularity of the field for conductors of vanishing thickness [52], the square of the magnetic field tangential to the metallization is not integrable. However, this can be repaired to achieve a good approximation of conductor loss by proper modification of the asymptotic behavior of the transform of the magnetic field [24], [25]. The idea behind this is that, except for the immediate vicinity of the conductor edges, the field does not change noticeably if a small, finite thickness is introduced. The modification may also be performed in the spatial domain if a strip-type problem prevails for which the original current density distribution is available in closed form. Also, longitudinal strip current or transverse slot voltage may be evaluated in

the space domain for the same reason. These quantities are often used in the calculation of characteristic impedances of strip and slot transmission lines [8], [9], [24], [34].

Similar advantageous properties as those described for the spectral-domain approach are shared to some degree by one of the methods recently applied to MIC's. This is named the differential-difference approach (DDA) here and is also called the method of lines [17], [53]–[55]. The fundamental similarity to the SDA formulation consists in the fact that it reduces the original Helmholtz equation (1) to a system of ordinary differential equations which can be solved explicitly. In contrast to the spectral-domain approach, the reduction in complexity and presumption for further analytical processing is achieved by discretization of the Helmholtz operator, writing, for example,

$$\left( \frac{\partial^2}{\partial z^2} + k_i^2 \right) f_i^{m,n} + \frac{f_i^{m-1,n} - 2f_i^{m,n} + f_i^{m+1,n}}{h_x^2} + \frac{f_i^{m,n-1} - 2f_i^{m,n} + f_i^{m,n+1}}{h_y^2} = 0. \quad (16)$$

This implies a two-dimensional finite-difference representation of the field for each plane of constant coordinate  $z$ , i.e., a mesh of points  $m, n$  with  $m=1 \dots M, n=1 \dots N$ . It describes a band-structured system of coupled ordinary differential equations with a total number of  $2MN$  unknowns for two scalar wave potentials. The system can be decoupled, i.e., brought into diagonal form, leading to the same number of discrete transformed potentials, say  $\bar{f}_i^{m,n}$  and  $\bar{g}_i^{m,n}$ . For these, the  $z$ -dependency in the layered MIC structure can be described in analytical form including the boundary conditions at the ground and top planes. For the associated discretized tangential electric field and current density in the plane of metallization, the boundary conditions are formulated pointwise. This cannot be performed in terms of the transformed quantities  $\bar{f}_i^{m,n}, \bar{g}_i^{m,n}$  and, therefore, requires a back-transformation into the original domain. As in the spectral-domain approach, the last step in the DDA procedure is the solution of a determinantal equation depending on one of the physical parameters  $p$  of the problem or the solution of a deterministic linear system of equations for prescribed values of  $p$ .

The method has been applied only to shielded structures so far, which is a consequence of the spatial discretization that makes it difficult to extend it to open regions. Several interesting similarities between the SDA and the DDA become plausible if one recalls that the application of finite integral transforms means a discretization in the spectral domain. Extension to open problems with the SDA is straightforward since fields of finite and infinite spatial support both have contributions over the infinite spectral domain. One of the advantages of the method of lines is that it exhibits a comparatively low numerical expense for the generation of each of the elements of the final matrix equation. In addition, it can in a very flexible way be used for the analysis of different conductor shapes and does not require a choice of expansion functions. On the other hand,

application of the DDA to three-dimensional field problems results in very high matrix orders. If, for example, in a strip-type problem, a rectangular shielding with ground plane  $F$  and conductor area  $F_{st}$  is assumed, the order of the final DDA matrix is approximately

$$Q = 2M \cdot N \cdot \frac{F_{st}}{F}. \quad (17)$$

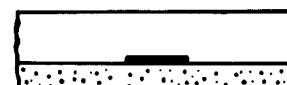
With growing complexity of the conductor shape, the spatial resolution  $M \cdot N$  has to be increased and the number of floating-point operations in the differential-difference approach is proportional to  $Q^3$ . About the same resolution is achieved by an SDA treatment using  $2MN$  Fourier coefficients, which, however, determines only the linear number of summations necessary to construct a matrix element. The order of the final SDA matrix is not directly related to  $M \cdot N$ , but mainly a question of the intelligent choice or systematic generation of expansion functions. It can be made extremely low, which makes the SDA a preferable technique for the repeated generation of MIC design information. From this point of view, it is an advantage that it allows the choice of expansion functions. Furthermore, the spectral-domain approach is specifically suited to analytical preprocessing and speedup measures as will be shown in the last section of this paper.

#### IV. SPECIFIC ASPECTS

In the analysis of MIC configurations by the spectral-domain approach three classes of structures have to be distinguished: shielded, covered, and open types. These are shown in Fig. 2 for the cross section of a single microstrip line. The shielded-type has been used extensively by contributors to the SDA in transmission-line and resonator problems. It presents a good description of real-life MIC structures only if radiation and surface-wave excitation from an adequate open structure are negligible. This applies, for example, for the technically used fundamental modes of printed strip and slot transmission lines under normal operating conditions [56]–[58] and to high- $Q$  resonators with properly chosen, not too large, shielding dimensions [14]–[16], [30], [31]. However, practical MIC shielding cases are usually large in dimensions compared to the enclosed circuit elements, with the exception of finlines and related millimeter-wave components [59], [60]. Therefore, care has to be taken in the interpretation of data derived from a shielded-type analysis if these shall be used for MIC design purposes. With respect to this point of view, the use of the covered, i.e., laterally open-type of analysis seems to present a better choice for the characterization of MIC structures in general. A cover shielding can always be specified in the design of a practical circuit and, therefore, taken into account properly. The assumption of lateral openness is believed to provide the most realistic one if design quantities have to be computed for general applicability in the CAD of MIC's or as a basis for the generation of mathematical models. Nevertheless, nearly all of the SDA contributions to the analysis of covered-type configurations neglect energy leakage into the lateral direction. They are equivalent, therefore, to shielded-type SDA



(a)



(b)



(c)

Fig. 2. Microstrip cross sections representing three different classes of MIC structures, (a) shielded, (b) covered, and (c) open type.

formulations with side walls removed left and right to infinity. Only recently, lateral energy leakage has been included in covered MIC analyses [40], [41], [61] and is considered a prerequisite to the description of dynamic coupling mechanisms. The open-type analysis of MIC structures, such as the one shown in Fig. 2(c), is applicable to nonradiating transmission-line modes, but mainly reserved to problems where radiation into free space is of dominant interest.

Using the shielded-type formulation, a systematic spectral-domain technique for the hybrid-mode characterization of MIC discontinuities and junctions has been developed by the author a few years ago [36], [62]. It is represented pictorially in the flow diagram of Fig. 3 to show how the SDA can be applied to derive design information in a very general way. The technique avoids the necessity of specifying sources and has, meanwhile, been applied successfully to a variety of strip- and slot-type problems [63]–[67]. It mainly refers to operating conditions where energy leakage into the volume field is not noticeable, but can, however, be extended to be valid without that restriction. The main idea behind the technique shown in Fig. 3 is a generalization of the Weissflock or tangent method [28] in conjunction with a three-dimensional SDA resonator analysis. This generalization can be performed and becomes practicable here because the total electromagnetic field and current density is available from the analysis which would not be accessible or practicable in an equivalent measurement situation. On top of the left column of Fig. 3, the physical  $n$ -port investigated is shown in a shielding box with the field volume subdivided into short-circuited transmission-line sections (stubs) of length  $l_i$  and the  $n$ -port junction. The circuit representation using scattering parameters is depicted on the right-hand side with the respective reference planes  $RP_i$ ,  $i = 1 \dots n$ . For

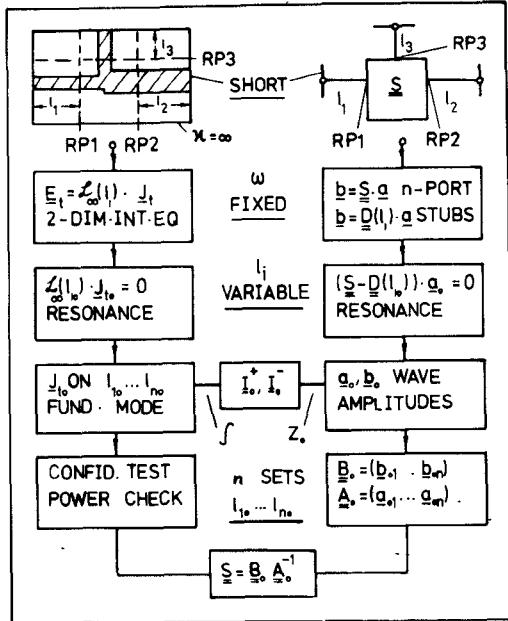


Fig. 3. A specific implementation of the SDA used for the frequency-dependent characterization of discontinuities in strip- and slot-type MIC's.

fixed operating frequency  $\omega$ , the configuration is analyzed in terms of successively interchanged resonant lengths  $l_i$  exactly  $n$  times. By introduction of precomputed strip current density distributions into the expansion functions used to describe  $\underline{J}_i$ , all the boundary conditions except those in the  $n$ -port region itself can be satisfied *a priori*. The resonant lengths and the stub current density amplitudes result from the numerical description of the  $n$  successive hypothetical resonator experiments. They are processed to obtain the complex amplitudes of the longitudinal strip currents or electric fields of the stubs. Then, using a power-related definition of characteristic stub impedance, the complex wave amplitudes associated with each of the  $n$  experiments are computed and assembled into matrices  $\underline{A}_0$  and  $\underline{B}_0$ . The scattering matrix of the investigated  $n$ -port results from this easily. As a confidence test for the validity of the results it is checked in parallel that the power balance for the lossless  $n$ -port is satisfied to a good approximation. The technique has the advantage of providing phase information which is stationary with respect to the current density and field distribution, respectively [36]. It has its limitations if radiation mechanisms in MIC's are involved to a noticeable degree.

To understand leakage mechanisms in MIC's, the mathematical structure of the spectral immittance matrices of (10) has to be considered. Independent of the degree of openness and the number of dielectric layers prevailing in a specific problem, the spectral impedances can always be written in the form [31], [34], [36], [38], [41]

$$\underline{\underline{Z}}(p) = \begin{bmatrix} k_x^2 Z_{FE} + k_y^2 Z_{FH} & k_x k_y (Z_{FE} - Z_{FH}) \\ k_x k_y (Z_{FE} - Z_{FH}) & k_y^2 Z_{FE} + k_x^2 Z_{FH} \end{bmatrix}$$

with

$$Z_{FE} = Z_{FE}(k_\rho^2, p) \quad Z_{FH} = Z_{FH}(k_\rho^2, p).$$

The admittance matrix for slot-type problems follows from inversion of (18) and of the impedance elements  $Z_{FE}$ ,  $Z_{FH}$ . It has exactly the same structure. Due to this duality, it is again sufficient to discuss the strip-type case for simplicity. The quantities  $Z_{FE}$  and  $Z_{FH}$  are the total LSM and LSE modal input wave impedances as seen into the medium below and above the plane of metallization. Thinking in terms of a transverse resonance approach [28], [29], therefore, makes clear that  $1/Z_{FE}$  and  $1/Z_{FH}$  have the properties of radial wave eigenfunctions in the layered circuit medium ( $1/Y_{FE}$  and  $1/Y_{FH}$  for slot-type problems). So, the elements  $Z_{FE}$  and  $Z_{FH}$  have poles for those values  $k_\rho = k_{\rho p}$  of the radial wavenumber which are propagation constants of the LSM and LSE modes in the inhomogeneous parallel-plate medium between the ground and top planes if the conductor metallization is not present and they represent surface waves for open-type structures [36], [41]. The maximum discrete value of  $k_{\rho p}^2$  corresponds to the dominant  $LSM_0$  radial wave in the circuit medium which is the main cause for dynamic coupling in MIC's since this has zero cutoff frequency. With

$$k_{xp} = k'_{xp} + jk''_{xp} = \pm (k_{\rho p}^2 - k_y^2)^{1/2} \quad (19)$$

the associated poles in the complex  $k_x$ -plane are all off the real  $k_x'$ -axis as long as  $k_y^2$  is larger than the value of  $k_{\rho p}^2$  of the  $LSM_0$  mode. Physically, this means, for example, that MIC transmission-line modes with propagation constants  $k_y$  larger than that of the  $LSM_0$  mode are nonradiating. This has already been discussed by Pregla in an early contribution [56]; however, his analysis has not been extended into the radiation region.

Higher order modes on covered and open MIC transmission lines do not generally exist in nonradiating form. Also, the respective MIC problems involving three-dimensional fields are always affected by energy leakage [40], [41] even if this may not be of practical concern at low frequencies. The SDA formulation can be extended in application to these cases by writing the scalar products of the final equations (14) in the form

$$(\underline{\underline{J}}_{tj}, \underline{\underline{Z}}(p) \underline{\underline{J}}_{tk})_x = \int_{C_x} \underline{\underline{Z}}(k_x, p) \underline{\underline{J}}_{tk}(k_x) \cdot \underline{\underline{J}}_{tj}(-k_x) dk_x \quad (20)$$

which is a consequence of Parseval's theorem [56] and by proper choice of the integration path  $C_x$ . The encountered immittance elements are meromorphic functions with respect to  $k_x$  in the covered case. The evaluation of (20) is achieved by residue calculus techniques [68]. In three-dimensional problems, in addition, integration in the proper related sheet of the complex  $k_y$  plane is involved. The simple principle of extension into the radiation region is a further fundamental advantage of the spectral-domain approach and allows rigorous treatment of complex MIC problems.

For the general rules of evaluating SDA integrals of the type (20) and a discussion of the physical background, a

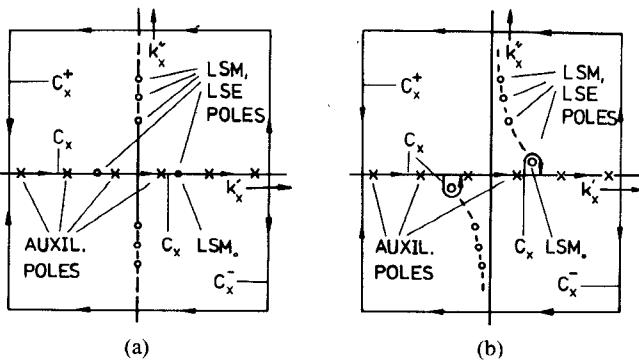


Fig. 4. Integration paths  $C_x$  in the SDA treatment of covered strip-type MIC problems, energy leakage (a) neglected and (b) taken into account, respectively.

covered transmission-line configuration such as that of Fig. 2(b) is considered. By introduction of the factor

$$1 = \frac{\exp(jck_x w)}{2\cos(ck_x w)} + \frac{\exp(-jck_x w)}{2\cos(ck_x w)}, \quad c > 1 \quad (21)$$

the integrals in (20) can be split up into a sum of two contributions for each of which the integration path  $C_x$  can be closed in the complex  $k_x$ -plane at infinity [39], [41]. This is shown in Fig. 4(a) for an analysis which does not include energy leakage, and in Fig. 4(b) where this mechanism is properly accounted for. In both cases, the zeros of the auxiliary cosine function in (21) introduce additional, non-physical poles onto the real  $k'_x$ -axis. According to the relation (19) and neglecting material loss, the LSM and LSE wave poles are located on the axes or not, depending on whether the leakage mechanism is incorporated into the solution (square of propagation constant  $k_y^2$  complex) or not ( $k_y^2$  a real number). The quantity  $w$  is a suitable normalization width. The positive real constant  $c$  is to some degree arbitrary and can be utilized for numerical check purposes and in convergence considerations. Integrating along the real  $k'_x$ -axis across the single poles artificially introduced by (21), Cauchy principal values are taken [68].

If, in Fig. 4(a), a nonradiating mode would be considered, i.e., with a propagation constant larger than that of the LSM radial wave, the  $LSM_0$  pole would be located on the imaginary  $k''_x$ -axis. In that case, the dominant contributions to the integrals in (20) would come from the discrete, regular set of auxiliary poles, say  $k_{xm}$ , on the real axis. With the constant  $c$  in the factor (21) chosen sufficiently large, the set of  $k_{xm}$  becomes very dense and the problem could be described in terms of this set alone. In the limit of  $c \rightarrow \infty$ , this is nothing else than numerical integration along the real axis of the  $k_x$ -plane. However, describing an MIC problem under radiation conditions, as actually prevailing with the pole locations of Fig. 4(a), this becomes more complicated. Numerical integration along the  $k'_x$ -axis and across the  $LSM_0$ -pole (Cauchy principle value) now means introducing a discrete standing plane contribution  $k_{xp}$  into the electric-field distribution [61]. This is equivalent to the presence of a lateral shielding far away from the MIC configuration, reflecting the radiated  $LSM_0$ -field. The same type of result is achieved if one applies a

transverse resonance approach to covered structures under operating conditions in the radiation region [57], [58].

The leaky character of higher order MIC transmission-line modes, discontinuities, and junctions of the covered-type are correctly described by the integration path shown in Fig. 4(b). This can be concluded from an investigation of the migration paths of the LSM and LSE wave poles in a slightly lossy dielectric medium [41], [61]. Those physical poles which in the lossless case of Fig. 4(a) are located on the real axis (here only the  $LSM_0$ -pole) are just below the  $k'_x$ -axis for a small dielectric tangent different from zero. They migrate across the real axis of the  $k_x$ -plane if an additional radiation mechanism is involved. Therefore, the original integration path  $C_x$  (the  $k'_x$ -axis) for nonradiating situations has to be distorted in the way indicated in Fig. 4(b). Otherwise, solutions would not pass over continuously into the radiation region of operation. As Pregla has already pointed out in his early study [56], the transition from one state of a solution to another, i.e., when the  $LSM_0$ -pole appears at the origin, does not present problems since the associated residues vanish then. The same arguments and integration path discussed here are valid also for the evaluation of integrals (scalar products) with respect to the  $k_y$ -variable in the SDA solution of three-dimensional field problems. However, depending on which integration is performed first, additional branch cuts have to be regarded either in the complex  $k_y$ - or  $k_x$ -plane. A broad and thorough treatment of the spectral-domain approach for a variety of representative leaky MIC problems has been elaborated in a recent dissertation by Boukamp [41].

## V. PROGRESS: 1975-1984

To round out the view given so far for the spectral-domain approach, a review is given of the improvements of the technique and the more important results achieved by its application during the last ten years. Emphasis is placed on frequency-dependent solutions since these become more and more important with the development of practical MIC's in the millimeter-wave region. Many of the contributions mentioned do not use the SDA in its pure form but deviate from it in the one step or another of the analysis. The reader may get an impression, therefore, that a high degree of flexibility is inherent in the details of the SDA formulation. The discussion is subdivided according to three different groups of MIC structures, namely transmission lines, resonators and antennas, and, finally, discontinuities.

Considering printed microwave and millimeter-wave transmission lines first, there has been a clear tendency since 1975 to treat this class of problems in a generalized way, allowing additional dielectric layers and more complicated strip and slot configurations [24], [26], [32], [34], [35], [59], [60], [69]-[78]. The inclusion of characteristic impedance data becomes standard in computer analysis programs, and also dielectric and conductor losses are considered frequently [24], [26], [34], [59], [60], [69], [72]-[78], [79]-[80]. This makes visible the beginning orientation of the SDA towards direct application in the design

of MIC's. Together with this trend, computer time and storage requirements for the analysis programs become a more important point of view [80]. However, from an appraisal of methods applied to the microstrip dispersion problem published by Kuester and Chang in 1979 [81], it can be concluded that the majority of respective computer packages at that time still involved some problems. There was significant progress, therefore, when not only the number of applications of the SDA increased further, but in addition some conceptual simplifications, modifications, and basic numerical considerations were reported. As elegant and simple concepts, for example, the transfer matrix approach [32] and the very similar spectral-domain immittance concept [35] have been presented. Also, El-Sherbiny [82], [83] provided interesting aspects to the mathematical and physical background of the SDA and applied a modified Wiener-Hopf technique in the final step of solution. Some of the rules to be regarded in order to obtain stable, accurate solutions and a unified treatment of shielded, covered, and open strip and slot structures have been reported by the author [34]. The introduction of finite conductor thickness into the SDA formalism is mainly the result of Kitazawa's work [21]–[23], [84]. Its effect on coplanar waveguide properties, for example, is an increase of guided wavelength and a decrease of characteristic impedance, respectively.

With growing experience in the use of the SDA, application of the technique shifted to more involved transmission-line problems. Coupled strip-slot structures have been studied by various authors with regard to coupler design and an extension of the range of characteristic impedances achievable in microstrip [22], [35], [47], [71], [77]. Also, an increasing portion of SDA work on transmission-line structures with anisotropic media has been reported. Borburgh [85], [86] seems to have been the first to apply the technique to microstrip on a magnetized ferrite substrate and related analyses followed [87], [88]. A variety of authors have treated printed transmission lines in single- and double-layered anisotropic dielectric media [89]–[92]. Only recently, slow-wave MIS coplanar waveguide has been studied with respect to MMIC application [93]. Beyond this, the computation of the stopband properties of several periodic structures by the spectral-domain approach has been reported [88], [94]–[96]. The last-mentioned reference also contains some numerical results on Podell-type microstrip couplers. As a further example for inhomogeneous structures, an analysis of tapered MIC transmission lines combining the SDA for uniform lines with coupled-mode theory has been presented [97]. Finally, a very efficient hybrid-mode spectral-domain approach for conductor arrays has been used by Jansen and Wiemer [98] in the design of MIC interdigital couplers and lumped elements on small computers.

Results achieved for resonators and antennas are considered together here since a large class of planar antennas makes use of resonating open patch elements. The information given on patch antennas, however, is by far incomplete, as the emphasis is placed on MIC's in this paper. The

first full-wave analyses of resonators concentrated on shielded structures and gave quite accurate results for the resonance frequencies and current density distributions of the open case if  $Q$ -factors were high and interaction with the volume field in the chosen shielding box low [30], [31]. Taking this into account, Jansen studied microstrip resonators of canonical and complicated shapes, with the latter described by a polygonal contour in terms of high-order finite-element polynomials for the current density [99].

Resonator shapes for which numerical results have been generated are rectangle, circular disk and ring, concentric coupled disk-ring and double-ring structure, stretched hexagon, and regular octagon. This work has recently been supplemented by Knorr [100] who analyzed a shielded short-circuited slotline resonator and by Sharma and Bhal [101], [102] who provided shielded-type results for the triangular shape and interacting rectangular microstrip structures.

Already by 1978, Pregla [43] had investigated open resonating microstrip rings including radiation using a Hankel transform and formulating the problem in terms of complex eigenfrequency. With increasing interest in microstrip antennas, further related analyses of circular shapes were performed in the years following [44]–[46], [103]–[105]. Itoh and Menzel presented a full-wave SDA treatment of open rectangular microstrip patches in 1981 [106] with clear emphasis on antenna applications. There is also direct antenna design work, for example, contributions by Bailey and Deshpande [107], [108] and by Newman *et al.* [109], which performs only part of the computational steps in the spectral domain. Numerical integration along the real axes in the spectral domain is the dominant choice in these papers; however, singularities near the integration path may cause problems (see, for example, Newman's remarks [109]). The first results for covered MIC geometries including the excitation of LSM and LSE waves in the layered circuit medium have been provided by Boukamp and Jansen [40], [41], [61]. The main intention of this research work was to study the mathematical and physical background and prepare the way for an extension of the SDA in application to dynamic MIC coupling problems. One of the practical results achieved in this context is, for example, that lateral leakage in MIC's can be minimized if the circuit cover height is chosen slightly lower than a value which would correspond to the onset of the first LSE mode.

The same covered-type spectral-domain approach has also been used to study the simplest case of a leaky microstrip discontinuity problem, i.e., the open end, with an excitation formulation [40], [41]. The motivation was again to provide a basis for the analysis of more complicated geometries. Interestingly, the numerical results in comparison with a resonator formulation indicate that there is a noticeable coupling effect between the open ends of half-wavelength resonators, such as those used, for example, in coupled line filters. The explanation is that, on the alumina substrate investigated, the distance between the respective open microstrip ends is small compared to

the wavelength of the involved  $LSM_0$ -mode. Beyond this very elementary but rigorous example, MIC discontinuities have been computed using the SDA only for the static limit [13] and by the frequency-dependent shielded-type SDA implementation outlined in the foregoing section [36], [62]–[67], [110]. Due to the three-dimensional electromagnetic fields and relatively complicated geometries involved, this sparsity of results prevails for other methods to an even larger extent. Systematic and quite extensive design data have been published for open-circuited-microstrip and suspended substrate lines, as well as short-circuited slots [36] and for the symmetrical and asymmetrical gap in microstrip and suspended substrate lines [63], [65]. Also, the inductive strip discontinuity in unilateral finlines, which is the related slot-type structure, has been treated [64]. Very recent work by Koster and Jansen provided a variety of microstrip impedance step data for use in MIC design [67], [110].

## VI. EFFICIENCY CONSIDERATIONS

The spectral-domain approach is a hybrid technique in the sense that it requires (and allows!) a certain amount of analytical preprocessing in order to achieve high computational efficiency for a specific problem or class of problems. One of its main disadvantages is the relatively high numerical expense which has to be spent to evaluate the coefficients of the final system of equations (14). These are improper integrals or infinite series with only moderate rate of convergence. The order of the final system, on the other hand, can be held extremely small compared to other techniques. This is achieved, for example, by regarding several criteria in the choice of the expansion functions [80]. Briefly summarizing, the set of expansion functions as a whole should be twice continuously differentiable in the interior of the region on which it is defined, so that it is in the domain of the original Helmholtz operator (1). Mathematical arguments and numerical experience indicate that this avoids the existence of spurious, nonphysical solutions [80]. Further, expansion functions in MIC problems should satisfy the edge condition, i.e., have the correct order of singularity at the boundary of the conductor metallization. This is a prerequisite to obtaining accurate solutions with a low number of terms or, equivalently, with a low order of the final system of equations. The set of functions used should be complete in order to allow convergence checks and investigation. It should be chosen with all the physical insight that is available for the specific problems, from static considerations, from idealizations such as the planar magnetic-wall waveguide model [19] and so on. The main rule is not to leave work to the computer for the evaluation of what is known in advance of the physical solution or can be obtained easier. This also implies the precomputation of expansion functions by a transmission-line SDA portion (two-dimensional fields) in computer programs for the SDA solution of three-dimensional field problems [36]. Finally, the use of static together with stationary precomputed information can provide a means to generate vector expansion functions based on the

continuity equation. The analytical and programming expense required on the side of the investigator may be considerable, which mirrors the hybrid character of the SDA. However, in the way outlined, very efficient CAD tools can be developed by its application.

To come to a quantification of the numerical expense associated with the SDA, the number of point operations which have to be performed in the solution of typical MIC problems is estimated. Also, the possibilities of reducing this figure shall be discussed. Let us assume a not too elementary MIC transmission-line case, in parallel, a resonator problem formulated in Cartesian coordinates. The final system of SDA equations (14) is dense and has to be generated repeatedly in the iterative localization of the zeros of its determinant as a function of the eigenvalue parameter  $p$ . Even with an intelligently chosen start value of  $p$ , this has to be done about 10 times. Under the assumption of a reasonable choice of expansion functions, the number of point operations necessary to obtain the numerical value of the final SDA determinant is usually small compared to the expense investigated for its generation. This is a consequence of the fact that the number of summations required to compute a single coefficient (integral or series) of (14) is typically much larger than the order of the system matrix. The latter may be

$$Q = 2 \cdots 10 \text{ and } Q = 4 \cdots 100 \quad (22)$$

for the transmission-line and resonator problem, respectively.

For example,  $Q = 4$  could apply to a simple, rectangular half-wavelength microstrip resonator [14], [15], [36]. The number of summations or discretization points to evaluate each single scalar product may be  $100 \cdots 500$  for the two-dimensional and  $100^2 \cdots 500^2$  for the three-dimensional case. In particular situations, this may be even higher [34] depending on the spectral distribution of the involved fields. The complexity of the admittance functions encountered depends only on the number of dielectric layers considered and may be characterized by a figure of at least  $10 \cdots 100$  point operations. On the whole, this amounts to a total count of point operations of about

$$TC = 2 \cdot 10^4 \cdots 25 \cdot 10^6 \text{ and } TC = 8 \cdot 10^6 \cdots 125 \cdot 10^{10} \quad (23)$$

for the two cases considered (symmetric SDA matrices). This looks quite high, particularly for the very right-hand side. However, one has to keep in mind that the spatial resolution assumed there is equivalent to a mesh of  $25 \cdot 10^4$  points in the plane of the MIC metallization. As a rough estimate, the matrix order in a respective DDA treatment would be  $Q = 5000$ , the matrix itself dense, and had to be processed repeatedly about 10 times.

A reduction of numerical expense in SDA solutions is achieved first by an optimization of the expansion functions. This is performed according to the outlined criteria and with some experience from a preliminary, crude version. It can be done with a relatively small amount of reprogramming and produces a typical speedup factor of

$5 \cdots 10$  for nonelementary two- and three-dimensional problems. Also, about a factor of 10 may be gained by choosing an excitation formulation instead of solving an eigenvalue problem which applies, however, only to the three-dimensional case. The estimated speedup results since the source formulation avoids repeated generation of the final system (14). An additional reduction in computer time can be obtained by splitting off asymptotic spectral contributions from the coefficients of (14) and integrating or summing up these by analytical techniques (a factor of 10). In eigenvalue problems, it is advisable to substitute the spectral immittances by accurate one-dimensional interpolants [36] and optimize CP-time at the cost of storage requirements [31] for that part of the computation which does not depend on the eigenvalue  $p$  (a factor of 10). SDA computer programs developed for regular industrial use in MIC design justify even more expense in analytical preprocessing. For these, the normal mode of application is the repeated solution of the same problem for several different operating frequencies. Therefore, a high speedup factor compared to the first solution can be achieved if this is employed to provide for the subsequent ones a very compact low-order set of expansion functions (tested by the author for the transmission-line problem described in [98]). Thus, average CP-time is further reduced by about a factor of 5. By a combination of such analytical measures, the total count of point operations may be brought down to

$$TC = 2 \cdot 10^3 \cdots 5 \cdot 10^5 \text{ and } TC = 8 \cdot 10^5 \cdots 25 \cdot 10^7 \quad (24)$$

which is hardly achievable by other techniques. However, great care has to be taken in properly designing the employed integration algorithms, i.e., choosing a correct spectral representation. This particularly effects cases where tight coupling is involved. For loose and multiple coupled situations, a sufficiently stable matrix inversion algorithm has to be chosen.

## VII. CONCLUSION

The spectral-domain approach allows an elegant and closed-form integral equation formulation for a broad class of MIC problems which is reduced by one dimension compared to the original field problem. It results in a particularly low-order linear system of equations and provides design-relevant parameters in both the spectral and the space domain. In so far as it may require a considerable amount of analytical preprocessing to achieve highest efficiency, it is a hybrid method. A preference of the SDA for MIC problems is to some extent confirmed by the fact that the majority of rigorous frequency-dependent MIC design information has been generated using this technique. The survey presented here further confirms this preference; however, the advice should be deduced from the discussion not to apply the SDA in a crude and schematic way.

## REFERENCES

- [1] I. N. Sneddon, *The Use of Integral Transforms*. New York: McGraw-Hill, 1972.
- [2] Tai Tsun Wu, "Theory of the microstrip," *J. Appl. Phys.*, vol. 28, pp. 299-302, 1975.
- [3] E. Yamashita and R. Mittra, "Variational method for the analysis of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 251-256, 1968.
- [4] E. J. Denlinger, "A frequency dependent solution for microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-19, pp. 30-39, 1971.
- [5] H. J. Schmitt and K. H. Sarges, "Wave propagation in microstrip," *Nachrichtentech. Z.*, vol. 5, pp. 260-264, 1971.
- [6] T. Itoh and R. Mittra, "Dispersion characteristics of slot lines," *Electron. Lett.*, vol. 7, pp. 364-365, 1971.
- [7] T. Itoh and R. Mittra, "Spectral-domain approach for calculating the dispersion characteristics of microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-21, pp. 496-499, 1973.
- [8] G. Kowalski and R. Pregla, "Dispersion characteristics of single and coupled microstrips," *Arch. Elek. Übertragung*, vol. 26, pp. 276-280, 1972.
- [9] M. K. Krage and G. I. Haddad, "Frequency-dependent characteristics of microstrip transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 678-688, 1972.
- [10] A. R. Van de Capelle and P. J. Luypaert, "Fundamental- and higher-order modes in open microstrip lines," *Electron. Lett.*, vol. 9, pp. 345-346, 1973.
- [11] T. Itoh and R. Mittra, "A technique for computing dispersion characteristics of shielded microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 896-898, 1974.
- [12] R. H. Jansen, "A modified least-squares boundary residual (LSBR) method and its application to the problem of shielded microstrip dispersion," *Arch. Elek. Übertragung*, vol. 28, pp. 275-277, 1974.
- [13] Y. Rahmat-Samii *et al.*, "A spectral-domain analysis for solving microstrip discontinuity problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 372-378, 1974.
- [14] T. Itoh, "Analysis of microstrip resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 946-951, 1974.
- [15] R. H. Jansen, "Shielded rectangular microstrip disc resonators," *Electron. Lett.*, vol. 10, pp. 299-300, 1974.
- [16] R. H. Jansen, "Computer analysis of edge-coupled planar structures," *Electron. Lett.*, vol. 10, pp. 520-522, 1974.
- [17] U. Schulz, "The method of lines—A new technique for the analysis of planar microwave structures" (in German), Ph.D. dissertation, Univ. of Hagen, West Germany, 1980.
- [18] R. Mittra, Ed., *Computer Techniques for Electromagnetics* (Int. Series of Monographs in El. Eng.). New York: Pergamon Press, 1973.
- [19] K. C. Gupta *et al.*, *Microstrip Lines and Slotlines*. Dedham, MA: Artech House, 1979.
- [20] K. C. Gupta and A. Singh, *Microwave Integrated Circuits*. New Delhi: Wiley Eastern Private Ltd., 1974.
- [21] T. Kitazawa *et al.*, "A coplanar waveguide with thick metal-coating," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 604-608, 1976.
- [22] R. H. Jansen, "Microstrip lines with partially removed ground metallization-theory and applications," *Arch. Elek. Übertragung*, vol. 32, pp. 485-492, 1978.
- [23] T. Kitazawa *et al.*, "Analysis of the dispersion characteristics of slot line with thick metal coating," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 387-392, 1980.
- [24] R. H. Jansen, "Fast accurate hybrid mode computation of non symmetrical coupled microstrip characteristics," in *Proc. 7th Eur. Microwave Conf.*, 1977, pp. 135-139.
- [25] W. Schumacher, "Current distribution in the ground plane of covered microstrip and the effect on conductor losses" (in German), *Arch. Elek. Übertragung*, vol. 33, pp. 207-212, 1979.
- [26] D. Mirshekhar-Syakhal and J. B. Davies, "Accurate solution of microstrip and coplanar structures for dispersion and for dielectric and conductor loss," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 694-699, 1979.
- [27] R. Pregla, "Determination of conductor losses in planar waveguide structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 433-434, 1980.
- [28] R. E. Collin, *Field Theory of Guided Waves*. New York: McGraw-Hill, 1960.
- [29] R. R. Harrington, *Time-Harmonic Electromagnetic Fields*. New York: McGraw-Hill, 1961.
- [30] R. H. Jansen, "Computer analysis of shielded microstrip structures" (in German), *Arch. Elek. Übertragung*, vol. 29, pp. 241-247, 1975.

[31] R. H. Jansen, "Numerical computation of the resonance frequencies and current density distributions of arbitrarily shaped microstrip structures" (in German), Ph.D. dissertation, Univ. Aachen (RWTH), West Germany, 1975.

[32] J. B. Davies and D. Mirshekar-Syakhal, "Spectral domain solution of arbitrary coplanar transmission line with multilayer substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 143–146, 1977.

[33] L. J. van der Pauw, "The radiation of electromagnetic power by microstrip configurations," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 719–725, 1977.

[34] R. H. Jansen, "Unified user-oriented computation of shielded, covered and open planar microwave and millimeterwave transmission-line characteristics," *IEE J. Microwaves, Opt., Acoust.*, vol. MOA-3, pp. 14–22, 1979.

[35] T. Itoh, "Spectral domain imittance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 733–736, 1980.

[36] R. H. Jansen, "Hybrid mode analysis of end effects of planar microwave and millimeter-wave transmission lines," *Proc. Inst. Elec. Eng.*, pt. H, vol. 128, pp. 77–86, 1981.

[37] A. S. Omar and K. Schünemann, "Space domain decoupling of LSE and LSM fields in generalized planar guiding structures," in *IEEE MTT Symp. Dig.*, 1984, pp. 59–61.

[38] L. J. Van der Pauw, "The radiation and propagation of electromagnetic power by a microstrip transmission line," *Philips Res. Rep.*, vol. 31, pp. 35–70, 1976.

[39] R. H. Jansen, unpublished results and computer program—SFPMIC—, "Source formulation approach to planar microwave integrated circuits," Univ. of Duisburg, West Germany, 1979/80.

[40] J. Boukamp and R. H. Jansen, "The high-frequency behaviour of microstrip open ends in microwave integrated circuits including energy leakage," in *Proc. 14th Eur. Microwave Conf.*, 1984, pp. 142–147.

[41] J. Boukamp, "Spectral domain techniques for the analysis of radiation effects in microwave integrated circuits" (in German), Ph.D. dissertation, Univ. of Aachen (RWTH), West Germany, 1984.

[42] T. Itoh and R. Mittra, "Analysis of a microstrip disk resonator," *Arch. Elek. Übertragung*, vol. 27, pp. 456–458, 1973.

[43] S. G. Pintzos and R. Pregla, "A simple method for computing the resonant frequencies of microstrip ring resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 809–813, 1978.

[44] W. C. Chew and J. A. Kong, "Resonance of the axial-symmetric modes in microstrip disk resonators," *J. Math. Phys.*, vol. 21, pp. 582–591, 1980.

[45] W. C. Chew and J. A. Kong, "Resonance of nonaxial symmetric modes in circular microstrip disk antenna," *J. Math. Phys.*, vol. 21, pp. 2590–2598, 1980.

[46] K. Araki and T. Itoh, "Hankel transform domain analysis of open circular microstrip radiating structures," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 84–89, 1981.

[47] D. Mirshekar-Syakhal and J. B. Davies, "Accurate analysis of coupled strip-fineline structure for phase constant, characteristic impedance, dielectric and conductor losses," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 906–910, 1982.

[48] I. V. Lindell, "Variational methods for nonstandard eigenvalue problems in waveguide and resonator analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1194–1204, 1982.

[49] G. J. Gabriel and I. V. Lindell, "Comments on Variational methods for nonstandard eigenvalue problems in waveguide and resonator analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 786–789, 1983.

[50] G. J. Gabriel and I. V. Lindell, "Further comments on Variational methods for nonstandard eigenvalue problems in waveguide and resonator analysis," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 474–476, 1984.

[51] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968.

[52] R. Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*. New York: Macmillan, 1971.

[53] S. B. Worm, "Analysis of planar microwave structures of arbitrary shape" (in German), Ph.D. dissertation, Univ. of Hagen, West Germany, 1983.

[54] H. Diestel and S. B. Worm, "Analysis of hybrid field problems by the method of lines with nonequidistant discretization," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 633–638, 1984.

[55] S. B. Worm and R. Pregla, "Hybrid-mode analysis of arbitrarily shaped planar microwave structures by the method of lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 191–196, 1984.

[56] G. Kowalski and R. Pregla, "Dispersion characteristics of single and coupled microstrips with double-layer substrates," *Arch. Elek. Übertragung*, vol. 27, pp. 125–130, 1973.

[57] H. Ermert, "Guided modes and radiation characteristics of covered microstrip lines," *Arch. Elek. Übertragung*, vol. 30, pp. 65–70, 1976.

[58] H. Ermert, "Guiding characteristics and radiation characteristics of planar waveguides," in *Proc. 8th Eur. Microwave Conf.*, 1978, pp. 94–98.

[59] H. Hofmann, "Dispersion of planar waveguides for millimeter-wave application," *Arch. Elek. Übertragung*, vol. 31, pp. 40–44, 1977.

[60] L. P. Schmidt and T. Itoh, "Spectral domain analysis of dominant and higher order modes in fin lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 981–985, 1980.

[61] J. Boukamp and R. H. Jansen, "Spectral domain investigation of surface wave excitation and radiation by microstrip lines and microstrip disk resonators," *Proc. 13th Eur. Microwave Conf.*, 1983, pp. 721–726.

[62] R. H. Jansen, "A new unified numerical approach to the frequency dependent characterization of strip, slot and coplanar MIC components for CAD purposes," Univ. of Duisburg, West Germany, Res. Rep. FB9/ATE, pp. 29–33, 1980/1981.

[63] R. H. Jansen and N. H. L. Koster, "A unified CAD basis for the frequency dependent characterization of strip, slot and coplanar MIC components," in *Proc. 11th Eur. Microwave Conf.*, 1981, pp. 682–687.

[64] N. H. L. Koster and R. H. Jansen, "Some new results on the equivalent circuit parameters of the inductive strip discontinuity in unilateral fin lines," *Arch. Elek. Übertragung*, vol. 35, pp. 497–499, 1981.

[65] N. H. L. Koster and R. H. Jansen, "The equivalent circuit of the asymmetrical series gap in microstrip and suspended substrate lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1273–1279, 1982.

[66] R. H. Jansen and N. H. L. Koster, "New aspects concerning the definition of microstrip characteristic impedance as a function of frequency," in *IEEE MTT-Symp. Dig.*, 1982, pp. 305–307.

[67] N. H. L. Koster and R. H. Jansen, "The microstrip discontinuity: A revised description," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, 1985.

[68] G. B. Arfken, *Mathematical Methods for Physicists*. New York: Academic Press, 1970.

[69] J. B. Knorr and K. D. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 541–548, 1975.

[70] N. Samardzija and T. Itoh, "Double-layered slot line for millimeter wave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 827–831, 1976.

[71] T. Itoh, "Generalized spectral domain method for multiconductor printed lines and its application to tunable suspended microstrips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 983–987, 1978.

[72] J. B. Knorr and P. M. Shayda, "Millimeter-wave fin line characteristics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 737–743, 1980.

[73] L. P. Schmidt *et al.*, "Characteristics of unilateral fin-line structures with arbitrarily located slots," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 352–355, 1981.

[74] D. Mirshekar-Syakhal and J. B. Davies, "An accurate unified solution to various fin-line structures, of phase constant, characteristic impedance, and attenuation," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1854–1861, 1982.

[75] D. Mirshekar-Syakhal, "An accurate determination of dielectric loss effect in MMIC's including microstrip and coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 950–954, 1983.

[76] A. K. Sharma and W. J. R. Hoefer, "Propagation in coupled unilateral and bilateral finlines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 489–502, 1983.

[77] W. Schumacher, "Computation of guided wavelengths of planar transmission lines in two- and three-layered media using a moment method" (in German), Ph.D. dissertation, Univ. of Karlsruhe, West Germany, 1979.

[78] W. Schuhmacher, "Hybrid modes and field distributions in unsymmetrical planar transmission lines" (in German), *Arch. Elek. Übertragung*, vol. 34, pp. 445-453, 1980.

[79] J. B. Knorr and A. Tufekcioglu, "Spectral domain calculation of microstrip characteristic impedance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 725-728, 1975.

[80] R. H. Jansen, "High speed computation of single and coupled microstrip parameters including dispersion, high-order modes, loss and finite strip thickness," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 75-82, 1978.

[81] E. F. Kuester and D. C. Chang, "An appraisal of methods for computation of the dispersion characteristics of open microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 691-694, 1979.

[82] A. M. A. El-Sherbiny, "Exact analysis of shielded microstrip lines and bilateral fin lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 669-675, 1981.

[83] A. M. A. El-Sherbiny, "Millimeter-wave performance of shielded slot-lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 750-756, 1982.

[84] T. Kitazawa and R. Mittra, "Analysis of finline with finite metallization thickness," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 1484-1487, 1984.

[85] J. Borburgh, "Theoretical investigation of the dispersion and field distribution of guided modes on a microstrip line with gyrotrropic substrate" (in German), Ph.D. dissertation, Univ. of Erlangen, West Germany, 1976.

[86] J. Borburgh, "The behaviour of guided modes on the ferrite-filled microstrip line with the magnetization perpendicular to the ground plane," *Arch. Elek. Übertragung*, vol. 31, pp. 73-77, 1977.

[87] Y. Hayashi and R. Mittra, "An analytical investigation of finlines with magnetized ferrite substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 495-498, 1983.

[88] C. Surawatpunya *et al.*, "Bragg interaction of electromagnetic waves in a ferrite slab periodically loaded with metal strips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 689-695, 1984.

[89] A. M. A. El-Sherbiny, "Hybrid mode analysis of microstrip lines on anisotropic substrates," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1261-1266, 1981.

[90] H. Lee and V. K. Tripathi, "Spectral domain analysis of frequency dependent propagation characteristics of planar structures on uniaxial medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1188-1193, 1982.

[91] T. Kitazawa and Y. Hayashi, "Propagation characteristics of striplines with multilayered anisotropic media," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 429-433, 1983.

[92] M. Horne and R. Marques, "Coupled microstrips on double anisotropic layers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 467-470, 1984.

[93] Y. Fukuoka *et al.*, "Analysis of slow-wave coplanar waveguide for monolithic integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 567-573, 1983.

[94] K. Ogusu, "Propagation properties of a planar dielectric waveguide with periodic metallic strips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 16-21, 1981.

[95] T. Kitazawa and R. Mittra, "An investigation of striplines and finlines with periodic stubs," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 684-688, 1984.

[96] F. J. Glandorf, "Numerical solution of the electromagnetic eigenvalue problem of periodically inhomogeneous microstrip lines" (in German), Ph. dissertation, Univ. of Duisburg, West Germany, 1982.

[97] D. Mirshekar-Syakhal and J. B. Davies, "Accurate analysis of tapered planar transmission lines for microwave integrated circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 123-128, 1981.

[98] R. H. Jansen and L. Wiemer, "Multiconductor hybrid-mode approach for the design of MIC couplers and lumped elements including loss, dispersion and parasitics," in *Proc. 14th Eur. Microwave Conf.*, 1984, pp. 430-435.

[99] R. H. Jansen, "High-order finite element polynomials in the computer analysis of arbitrarily shaped microstrip resonators," *Arch. Elek. Übertragung*, vol. 30, pp. 71-79, 1976.

[100] J. B. Knorr, "Equivalent reactance of a shorting septum in a fin line: Theory and experiment," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1196-1202, 1981.

[101] A. K. Sharma and B. Bhat, "Analysis of triangular microstrip resonators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 2029-2031, 1982.

[102] A. K. Sharma and B. Bhat, "Spectral domain analysis of interacting microstrip resonant structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 681-685, 1983.

[103] K. Kawano and H. Tomimuro, "Spectral domain analysis of an open slot ring resonator," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 1184-1187, 1982.

[104] K. Araki *et al.*, "A study on circular disk resonators on a ferrite substrate," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 147-154, 1982.

[105] S. M. Ali *et al.*, "Vector Hankel transform analysis of annular-ring microstrip antenna," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 637-644, 1982.

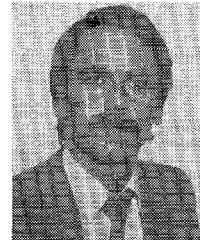
[106] T. Itoh and W. Menzel, "A full-wave analysis method for open microstrip structures," *IEEE Trans. Antennas Propagat.*, vol. AP-29, pp. 63-68, 1981.

[107] M. C. Bailey and M. D. Deshpande, "Integral equation formulation of microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 651-656, 1982.

[108] M. D. Deshpande and M. C. Bailey, "Input impedance of microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-30, pp. 645-650, 1982.

[109] E. H. Newman *et al.*, "Mutual impedance computation between microstrip antennas," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 941-945, 1983.

[110] N. H. L. Koster, "Frequency-dependent characterization of discontinuities in planar microwave transmission lines" (in German), Ph.D. dissertation, Univ. of Duisburg, West Germany, 1984.



**Rolf H. Jansen** (M'75-SM'84) was born in Cologne, West Germany, in 1946. He received the M.S. and Ph.D. degrees, both in electrical engineering, from RWTH Aachen in Aachen, West Germany, in 1972 and 1975, respectively. His M.S. thesis was on large-signal modeling of bipolar transistors for IC design application and his Ph.D. thesis was on the numerical analysis of arbitrarily shaped microstrip structures.

From 1972 to 1976, he was a Research Assistant at the RWTH Aachen microwave laboratory. From 1976 to 1979, he was a Senior Research Engineer in the same laboratory, working on the frequency-dependent characterization of planar structures and the CAD of microwave circuits, and was also responsible for the thin-film technology of the laboratory. Since 1977, he has been a scientific consultant and avocational staff member of Standard Elektrik Lorenz AG, SEL, Pforzheim, West Germany, in the Radio Communication Division. Since 1979, he has been a Professor of Electrical Engineering at the University of Duisburg, engaged in teaching and research in electromagnetic theory, in the microwave field, and the computer-aided design of microwave broad-band components, such as FET amplifiers and mixers. For the year 1981-1982, he was a full-time staff member of the Radio Communication Division of SEL, Pforzheim. He was responsible for the Introduction and testing of microwave CAD tools combined with several hardware projects in a production-oriented industry environment. He is the author and coauthor of about 50 technical papers in the microwave and CAD field. He is the cofounder and Scientific Counsellor of the MCAD Software + Design Corporation, Aachen, West Germany.

Dr. Jansen is a member of the Editorial Board of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES. Also, he is a member of the IEEE Standard Committee on Planar Transmission Lines (P 1004) and of the Microwave Field Theory Committee (MTT-15). He is a member of VDE, the German association of electrical engineers and was the recipient of the Outstanding Publications Award in 1979 of the German Society of Radio Engineers.